



RESEARCH DEPARTMENT

REPORT

CARFAX:
an example of an f.m. signal
generated using digital techniques

P. Shelswell, M.A., C.Eng., M.I.E.E.

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DIGITAL TECHNIQUES**
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Summary

This report describes a method of generating frequency-modulated signals using digital techniques. The digital process introduces little distortion and provides a stable centre frequency. The drive for a "CARFAX" transmitter is described as an example of its application.

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List of symbols

A	a constant
$f(t)$	modulating signal
$f_a(t)$	modulating signal in analogue form
$f_d(n\tau)$	modulating signal in digital form
f_s	sampling frequency
$J_n(\beta)$	Bessel function according to usual convention
K_{FM}	deviation of f.m. signal
n	number of sampling instants passed
N_p	noise power
$s(t)$	signal as a time function
$s_a(t)$	signal as a time function in analogue form
$s_d(n\tau)$	signal as a time function in digital form
t	time
β	modulation index
Δ	interval between quantization levels
Δf	bandwidth of the signal
θ_n	phase of signal w.r.t. carrier at n^{th} sampling instant
τ	sampling period
ω	angular frequency
ω_c	angular frequency of carrier
ω_m	angular frequency of modulation
ω_i	instantaneous angular frequency

1. Introduction

This report describes a means of generating frequency-modulated signals using digital techniques. The method could be used in principle, for stereophonic sound transmissions, but was first developed for "CARFAX"¹. It is, in principle, reliable and free from many forms of distortion.

Present drives for f.m. transmitters are analogue and prone to distortion. Care is needed to design a frequency modulator which has a linear relationship between input voltage and output frequency. Also the centre frequency and deviation are prone to drift. These imperfections and distortions can only be removed or avoided by an increase in the complexity of the drive.

An undistorted f.m. signal can be generated by digital techniques. The advantages include a stable centre frequency, a precisely determined deviation, and a linear relationship between input voltage and output frequency. In addition, digital devices are more immune to interference than analogue devices. This could be beneficial at transmitting sites with high radio-frequency fields.

The aim of this report is to outline the theory of a digital f.m. drive, and then explain how the theory is applied to the "CARFAX" drive.

2. General theory

The drive generates an f.m. signal using digital techniques. This involves calculating the amplitude of the signal at fixed time intervals. A digital-to-analogue converter then changes the signal from digital to analogue form for transmitting. The calculation of the signal's amplitude uses an algorithm derived from the standard analogue equations for frequency modulation.

The analogue equations for frequency modulation are well known. The instantaneous frequency is defined to be proportional to the input signal $f_a(t)$.

$$\omega_i = \omega_c + K_{FM} f_a(t) \quad (1)$$

This leads to an expression for the analogue signal

$$s_a(t) = \sin [\omega_c t + K_{FM} \int f_a(t) \cdot dt] \quad (2)$$

All the parameters on the right hand side of equation (2) are either defined or known variables. The signal is thus defined.

The digital algorithm is found by replacing the continuous time variables in equation (2) by a discrete variable $f_d(n\tau)$. The continuous variable t becomes the sampled variable $n\tau$, and integration becomes summation.

Hence,

$$s_d(n\tau) = \sin [n\omega_c \tau + K_{FM} \sum_{r=0}^n f_d(r\tau)] \quad (3)$$

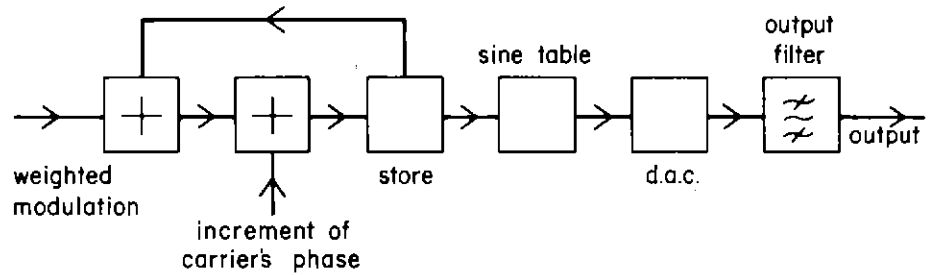
This could be used to calculate the signal directly, but the algorithm becomes more simple when equation (3) is rewritten.

$$s_d(n\tau) = \sin [\theta_n] \quad (4)$$

$$= \sin [\theta_{n-1} + \omega_c \tau + K_{FM} \tau f_d(n\tau)] \quad (5)$$

Equation (5) gives the phase of the sinusoid in terms

Fig. 1 - Schematic of the circuit needed to generate an f.m. signal using digital techniques.



of its previous phase and a known increment. The amplitude of the sinusoid can then be found using a memory containing a sine table.

The circuit which generates a signal using the digital algorithm is shown in Fig. 1. This is a realization of equation (5). The stored phase of the sinusoid is increased every clock period by an amount corresponding to the modulation and the change in the carrier. A memory then provides the sine of the angle. At this point the f.m. signal has been generated but is still in digital form. It only remains to convert the digital signal to analogue form and remove the harmonics of sampling frequency. The signal is then ready for use in the main transmitter.

3. Design theory

There are two major parameters which must be defined for a digital system — the sampling frequency and the number of quantizing levels.

3.1 Sampling frequency

The rules governing the sampling of signals are well known. To avoid aliasing, the signal must usually be band-limited and sampled at a frequency at least twice the maximum frequency component in the signal. (This is the Nyquist limit).

Sub-Nyquist sampling is possible for some band-limited signals which are modulated onto a carrier. If the sampling frequency is chosen suitably there is no interference between the wanted signal and the alias components. This is shown in Fig. 2. (The idea is the same as sub-Nyquist sampling of video signals). One disadvantage of this occurs when the digital signal is converted to analogue. Instead of producing impulses, the hardware usually uses a sample and hold mechanism. This adds a weighting of $(f_s/\pi f) \sin(\pi f/f_s)$ to the spectrum of Figure 2, giving low power levels at signal frequencies approaching the sampling frequency. Little power is lost when the sampling frequency is above the Nyquist limit, so a high sampling frequency is preferable.

It is difficult to specify the Nyquist limit for an f.m. signal which has been generated digitally. The theoretical representation of a sampled signal has an infinite bandwidth. If we consider the modulation, this will have a spectrum comprising the baseband component and additional components repeating at multiples of sampling frequency. When this signal is frequency modulated onto a carrier, Carson's rule indicates that we need an infinite bandwidth. Hence, the Nyquist limit appears meaningless at first sight. Sometimes the energy in the high frequency components is negligible. This is the case with "CARFAX". The f.m. signal can then be assumed to be bandlimited giving a Nyquist limit. The case where this assumption is not valid will not be considered here. When it is valid, the sampling frequency becomes limited by

$$f_s > \frac{1}{\pi} (\omega_c + K_{FM} + \omega_m) \quad (6)$$

(This is twice the upper frequency as determined by Carson's rule).

3.2 Accuracy of computation

The accuracy of each stage of computation is

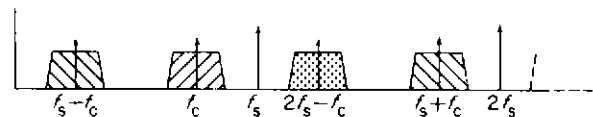
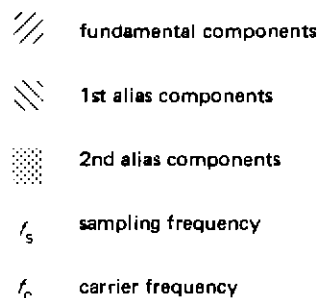


Fig. 2 - The effect of sub-Nyquist sampling of a narrowband signal



set by the quality of the output. This is specified in terms of noise and distortion. Once the output accuracy has been decided, the accuracy in the other stages follows logically. The problem is to relate the permissible noise and distortion at the output to the number of quantizing levels. The usual assumptions are not always valid.

Quantizing errors in the output signal can resemble distortion as well as noise. Most digital signals have statistics which cause quantizing errors to resemble white noise with an r.m.s. value of $\Delta^2/12$.² An important assumption in the derivation of this expression is that there is an equal probability of finding the unquantized version of the signal anywhere within one quantum step. With many wide deviation systems this is true and the noise level is readily defined. When there is little or no modulation this assumption is not necessarily true. There is then a significant correlation between the error signals for nearby samples. This alters the autocorrelation function of the error signal and, hence, the noise spectrum*, although it is difficult to say by how much without choosing a specific signal. In most cases the high frequency noise will be attenuated most. Sometimes, when the carrier frequency and the clock frequency are locked together, the autocorrelation function of the error signal can tend towards a series of impulses at carrier frequency. This gives a spectrum with strong components at carrier frequency and its harmonics. The error then is more like distortion than noise. All we can say about the power in this error signal is that it can lie anywhere between zero and $\Delta^2/4$; exactly where will depend on the signal.

If the f.m. signal is band-limited some of the quantizing noise and distortion may fall out of band. For the case of random white noise, the noise power will be proportional to the bandwidth. In the limit, though, the noise does not necessarily reduce to zero as the bandwidth reduces to zero. A bandwidth restriction on the signal leads to slow changes of instantaneous frequency. If the sampling frequency is locked to carrier this can lead to a repetitive error signal, as mentioned in the last paragraph. The associated line spectrum may have most of its power within the passband. As a form of distortion, this error may have a different effect from that of random noise. The simple approximation of noise power being proportional to bandwidth must, therefore, be used with care.

Having specified the noise and distortion which we can tolerate, the quantizing accuracy can

* The power density spectrum is the Fourier transform of the autocorrelation function.

be specified. If the signal is wideband with wide deviation, the noise power will be given by

$$N_p = \frac{\Delta^2}{12} \cdot \frac{\Delta f}{f_s} \quad (7)$$

For most f.m. signals the signal to noise ratio after demodulation is better than that before demodulation. This effect is well known and is reported in most textbooks on frequency modulation. If the error signal resembles distortion, the problem must be tackled specifically.

Once the output accuracy has been specified, the accuracy in the rest of the circuits follows naturally. This often entails working to a much greater accuracy within the drive.

4. The theory applied to "CARFAX"

The "CARFAX" drive is a practical application of the method. "CARFAX" uses both frequency modulation and amplitude modulation, and so needs a stable centre frequency in addition to the f.m. capability. This is difficult to provide using a conventional analogue system. The digital drive meets all the specifications easily.

4.1 Specification

1. Carrier

Centre frequency	526.5 kHz \pm 10 Hz
Drift	over 3 months and between 0° C and 45° C.

2. F.M. (provided by the drive)

Deviation	2 kHz \pm 5% (for sine waves)
Modulation	(a) 200 Hz sine wave (stop tone). (b) 125 Hz sine wave (start tone) (c) low frequency noise with 400 Hz deviation.

3. A.M. (applied in the output stages of the transmitter)

Noise	54 dB unweighted below 100% a.m.
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4.2 Sampling frequency

The sampling frequency is fairly arbitrary. Aliasing is negligible at sampling frequencies above the Nyquist limit as defined in equation (6).

A sampling frequency of four times carrier frequency is convenient and simplifies the hardware. For a sampling frequency of 2.106 MHz, the increment to the carrier's phase is 90° every clock period. If the phase is measured in binary fractions of 360° this addition is easy to perform. There are no problems of aliasing either.

Sub-Nyquist sampling is possible without incurring aliasing problems because of the low bandwidth compared with the carrier frequency. This allows a range of sampling frequencies up to just below twice carrier frequency. There will be occasional gaps in this range to prevent aliasing. The gaps are more numerous as the sampling frequency drops.

Sub-Nyquist sampling has no particular advantages for "CARFAX". There is no real need for a low sampling frequency. Any advantage gained is balanced by the shaping of the spectrum on conversion from digital to analogue form, which needs careful compensation.

The sampling frequency in the "CARFAX" drive was therefore chosen as 2.106 MHz.

4.3 Quantizing accuracy

The number of quantizing levels is not easy to define. This is because there are two different types of modulation which are transmitted at different times. The tones which control the receivers are radiated using frequency modulation, whilst the messages are radiated using amplitude modulation.

When the message is being transmitted, the quantizing errors resemble distortion of the carrier. The sampling frequency is locked to carrier frequency. The error signal, therefore, has a line spectrum with components at multiples of carrier frequency. The component at carrier frequency will alter the phase and amplitude of the main signal by a small, fixed amount. Because it is small and fixed it is nearly impossible to detect, and so is negligible.

When control tones are transmitted, the error signal resembles white noise if there are enough quantizing levels. This will only happen if the average change in the unquantized signal between

samples is of the same order as a quantization step. If it is not, then the noise spectrum once again tends towards a series of impulses. We know the rate of change of phase from equation (5). Hence, we know the average change in amplitude. This indicates that an accuracy of between 9 and 10 bits is desirable to ensure that the noise is white. An accuracy of ten bits is preferable as there are many d.a.c.s working to this standard. The carrier-to-noise ratio is then 82 dB for a bandwidth of ± 5 kHz. This is low enough not to affect any a.m. signal. The noise on the f.m. tones after demodulation is predominantly the quantizing noise present before modulation.

These arguments show that when the quantizing accuracy at the output is 10 bits, the quantizing errors are insignificant. Neither the message nor the control tones are affected seriously by quantization.

4.4 Implementation

The drive for "CARFAX" is a realization of the circuit of Fig. 1. The modulating signal is also generated digitally using a principle identical to that used to generate the main signal.

All the logic circuits use TTL. This is cheap and readily available. It is fast enough to work at a clock frequency of about 2 MHz and its low impedance makes it immune to radio frequency interference.

The circuits for the adders and stores are all conventional. These can be found in most manufacturers' TTL catalogues.

The sine table is an electronic version of a book of sine tables. Values of $\sin\theta$ are stored in a programmable read only memory (PROM) at coarse intervals in the first quadrant. Intermediate values are obtained using another PROM containing a fine difference. This is added to the coarse value to give an accurate value for the sine. Simple arithmetic gives values for θ in the other three quadrants.

The modulating signal is also generated by digital techniques. The two sine waves are generated using a memory containing a sine table. The phase at the input to the table is increased by a constant amount each clock period. The output is therefore a sampled and quantized sine wave. The low-frequency random noise is designed to reduce destructive interference between transmitters. This is accomplished by switching from 400 Hz above carrier frequency to 400 Hz

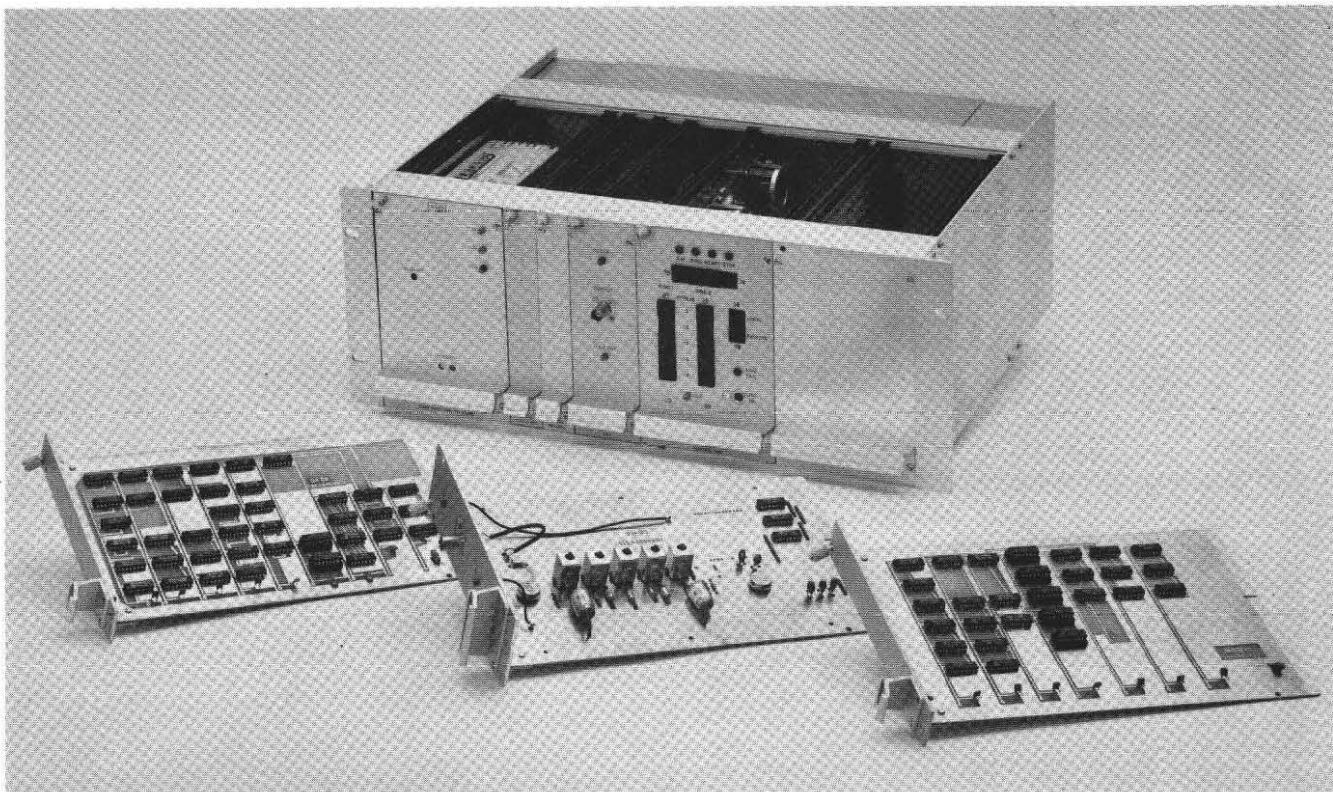


Fig. 3 - The "CARFAX" drive

below in a pseudo-random sequence using a clock rate of about 70 Hz.

The whole modulator fits on three printed circuit boards. Fig. 3 is a photograph of the drive, including power supplies and control circuits. The three principal boards are duplicated in front of the complete drive.

4.5 Output from the drive

The drive produces plain carrier or carrier with one of the three types of modulation required. The spectra of these are shown in Figs. 4a and b, 5, 6 and 7.

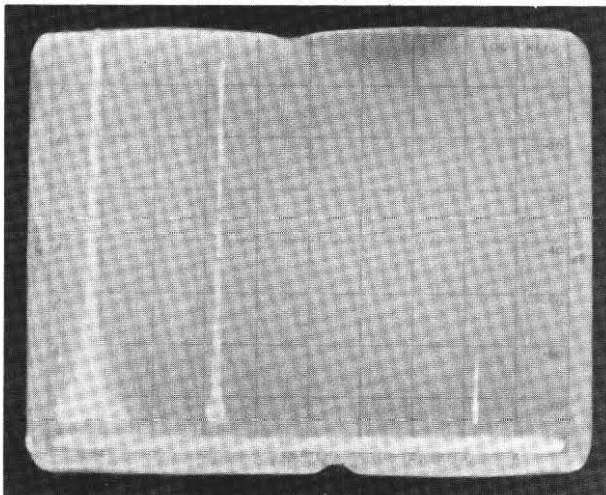
The carrier meets the normal broadcast standards. It has a stable frequency, the only drift being a direct result of the drift in the clock. F.M. noise is negligible, being impossible to measure on a standard spectrum analyser. A.M. noise is present, but in the form of coherent tones at multiples of carrier frequency. The components above carrier frequency are removed by filtering, whilst the component at carrier frequency is constant and, therefore, not noticeable.

The spectra for the start and stop signals are shown in Figures 5 and 6. The notable features of

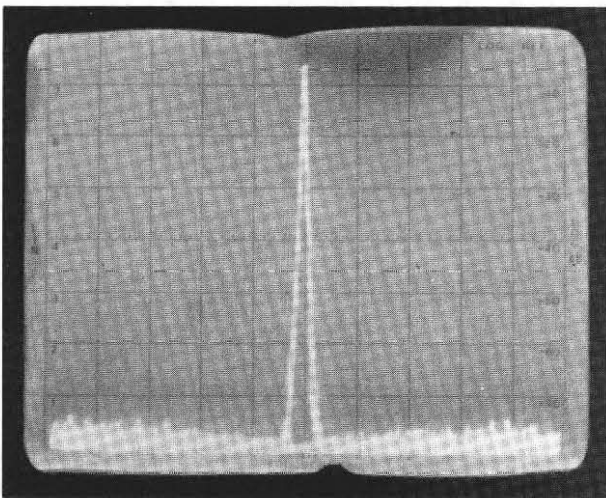
the spectra are their symmetry and predictability, showing that the modulation process is highly linear and accurate. Any non-linearity would show up as asymmetry. In these spectra there is no visible asymmetry, from which we infer a highly linear process. The levels of the spectral components are calculable from the modulation indices. The peak deviation is 1996.4 Hz and the modulating frequencies are 125.025 Hz and 199.965 Hz. This gives modulation indices (β) of 15.968 and 9.984. The level of the n^{th} spectral component is then $J_n(\beta)$. The levels of the smaller components are sensitive to β . By comparing their levels with theoretical values over a small range of β , we deduce that the modulation index is accurate to better than 0.1%.

The f.m. spectrum of the pseudo-random sequence is shown in Fig. 7. The modulation consists of a box-car waveform modulating the carrier with a deviation of 400 Hz. As the signalling frequency is low, the stationary frequency theory leads us to expect major frequency components at ± 400 Hz. These are present, but there are many others. These are predictable, but the analysis is complicated. Interested readers will find the method reported by Bennett and Rice³.

The drive, therefore, provides all the signals we want with all the relevant parameters accurately



(a)



(b)

Fig. 4 - Spectrum of the "CARFAX" carrier

- | | |
|------------------------------|----------------------------|
| (a) horizontal = 200 kHz/div | (b) horizontal = 1 kHz/div |
| c.f. = about 800 kHz | c.f. = carrier frequency |
| vertical = 10 dB/div | vertical = 10 dB/div |

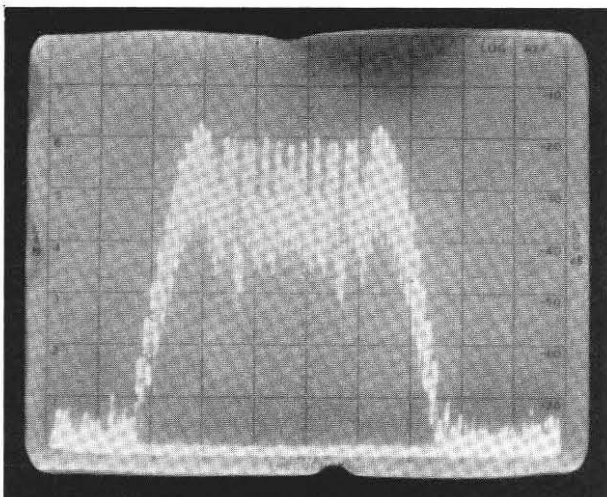


Fig. 5 - Spectrum of the "CARFAX" start tone

horizontal = 1 kHz/div
vertical = 10 dB/div

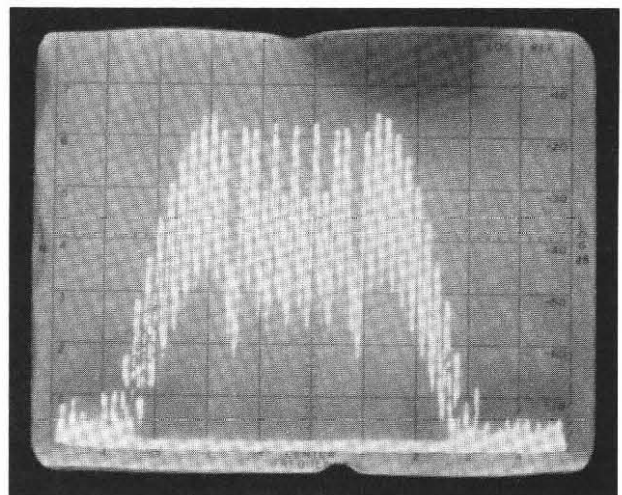


Fig. 6 - Spectrum of the "CARFAX" stop tone

horizontal = 1 kHz/div
vertical = 10 dB/div

defined. It is accurate and stable.

5. Conclusions

This report has described a method of frequency modulation. Using digital techniques, f.m. waveforms can be generated which are stable and accurate.

The theory has been applied to drive for "CARFAX" transmitters. The experimental results obtained from the "CARFAX" drives are in good agreement with theory.

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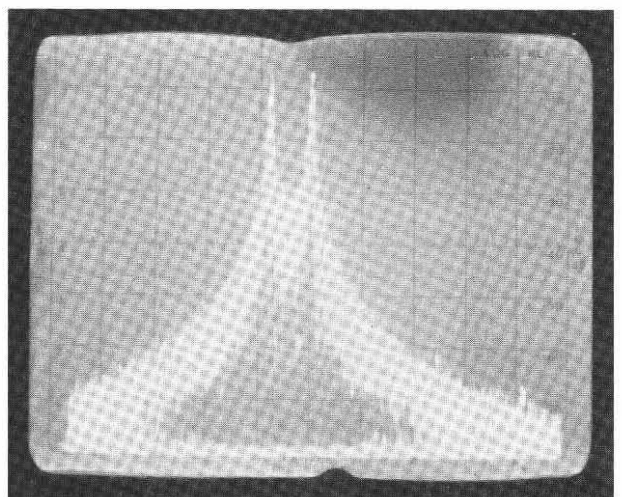


Fig. 7 - Spectrum of the "CARFAX" ring mode

horizontal = 1 kHz/div
vertical = 10 dB/div

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